Laplace Transform for impulse function:-

$$F(t) = \delta(t)$$

$$F(s) = \int_0^\infty f(t) \cdot e^{-st}\, dt = 1$$

EX Find the Laplace transform for $f(t) = 2\delta(t)$

$$F(s) = 2$$

Laplace transform for haperabolic function?

$$f_i(t) = \sinh at \Rightarrow F(s) = \frac{a}{s^2 - a^2}$$

$$f(t) = \cosh at \Rightarrow F(s) = \frac{s}{s^2 - a^2}$$

Initial and Final Values :-

The initial and final value properties allow to find the initial value $f(0)$ and the final value $f(\infty)$ of $f(t)$ directly from its Laplace transform $F(s)$.

$$f(0) = \lim_{s \to \infty} s\,F(s)$$

$$f(\infty) = \lim_{s \to 0} s\,F(s)$$

EX Find $f(0)$ for $f(t) = e^{-2t} \cos 10t$?

$$F(s) = \frac{s+2}{(s+2)^2+100}$$

$$F(0) = \lim_{s \to \infty} \left(\frac{s^2+2s}{(s+2)^2+100} \right)$$

$$F(0) = \frac{s^2+2s}{s^2+4s+4+100}$$

$$= \frac{\frac{s^2}{s^2} + \frac{2s}{s^2}}{\frac{s^2}{s^2} + \frac{4s}{s^2} + \frac{4}{s^2} + \frac{100}{s^2}}$$

$$= \frac{1+0}{1+0+0+0} = 1$$

EX Find initial value for $f(t) = e^{-2t} \sin 5t$.

$$F(s) = \frac{5}{(s+2)^2+25}$$

$$f(\infty) = \lim_{s \to 0} \frac{5s}{s^2+4s+4+25}$$

$$= \frac{0}{25} = 0$$

Laplace inverse :-

1- $\mathcal{L}^{-1}\frac{K}{S} = K\,u(t)$

EX $\mathcal{L}^{-1}\frac{2}{S} = 2\,u(t)$

2- $\mathcal{L}^{-1}\frac{K}{S \mp r} = K\,e^{\pm rt}$

EX $\mathcal{L}^{-1}\frac{2}{S+3} = 2\,e^{-3t}$

3- $\mathcal{L}^{-1}\frac{K}{S^r} = \frac{(-1)^r * K * t^{r-1}}{(r-1)!}$

EX $\mathcal{L}^{-1}\frac{6}{S^4} = \frac{(-1)^4 * 6 * t^3}{6} = t^3$

4- $\mathcal{L}^{-1}R = R\,\delta(t)$

EX $\mathcal{L}^{-1}5 = 5\,\delta(t)$

5- $\mathcal{L}^{-1}\frac{K}{(S \pm a)^r} = \frac{(-1)^r * K * t^{r-1}\,e^{\mp at}}{(r-1)!}$

EX $\mathcal{L}^{-1}\frac{6}{(S+2)^3} = \frac{(-1)^3 * 6 * t^2 * e^{-2t}}{2} = -3t^2\,e^{-2t}$

6- $\mathcal{L}^{-1}\frac{a}{S^2+a^2} = \sin at$

EX $\mathcal{L}^{-1}\frac{3}{S^2+9} = \frac{3}{\sqrt{2}}\,\sin\sqrt{2}\,t$

7- $\mathcal{L}^{-1} \dfrac{s}{s^2+a^2} = \cos at$

EX $\mathcal{L}^{-1} \dfrac{s}{s^2+5} = \cos\sqrt{5}\, t$

8- $\mathcal{L}^{-1} \dfrac{s-b}{(s\mp b)^2+a^2} = e^{\pm bt} \cos at$

EX $\mathcal{L}^{-1} \dfrac{s-2}{(s-2)^2+3} = e^{2t} \cos\sqrt{3}\, t$

9- $\mathcal{L}^{-1} \dfrac{a}{(s\mp b)^2+a^2} = e^{\pm bt} \sin at$

EX $\mathcal{L}^{-1} \dfrac{3}{(s-5)^2+2} = \dfrac{3}{\sqrt{2}} e^{-5t} \sin\sqrt{2}\, t$

10- $\mathcal{L}^{-1} F(s) \cdot e^{-rs} = F(t) \cdot u(t-r)$

EX $\mathcal{L}^{-1} \dfrac{2s \cdot e^{-3s}}{s^2+4} = 2\cos 2t\; u(t-3)$

11- $\mathcal{L}^{-1} \dfrac{s}{s^2-a^2} = \cosh at$

EX $\mathcal{L}^{-1} \dfrac{s}{s^2-5} = \cosh\sqrt{5}\, t$

12- $\mathcal{L}^{-1} \dfrac{a}{s^2-a^2} = \sinh at$

EX $\mathcal{L}^{-1} \dfrac{3}{s^2-4} = \dfrac{3}{2} \cdot \sinh 2t$

13- $\mathcal{L}^{-1} \dfrac{K}{(s+r_1)(s+r_2)(s+r_3)}$ (type one)

there are three theorems to find $\mathcal{L}^{-1}$.

__EX__ $\mathcal{L}^{-1} \dfrac{5}{(s+5)(s+2)}$

1) By partial fraction

$$\frac{5}{(s+5)(s+2)} = \frac{A}{s+5} + \frac{B}{s+2}$$

$$A = \lim_{s \to -5} \frac{5}{\cancel{(s+5)}(s+2)} \cancel{(s+5)} \Big|_{s=-5} = -\frac{5}{3}$$

$$B = \lim_{s \to -2} \frac{5}{(s+5)\cancel{(s+2)}} \cancel{(s+2)} \Big|_{s=-2} = \frac{5}{3}$$

$$\mathcal{L}^{-1} \frac{5}{(s+5)(s+2)} = -\frac{5}{3} \mathcal{L}^{-1} \frac{1}{s+5} + \frac{5}{3} \mathcal{L}^{-1} \frac{1}{s+2}$$

$$= -\frac{5}{3} e^{-5t} + \frac{5}{3} e^{-2t}$$

2) By algebric method :-

$$\frac{5}{(s+5)(s+2)} = \frac{A}{s+5} + \frac{B}{s+2}$$

$$\frac{5}{(s+5)(s+2)} = \frac{As + 2A + Bs + 5B}{(s+5)(s+2)}$$

$$5 = As + Bs + 2A + 5B$$

$0 = A + B \Rightarrow A = -B$ — (1) Coefficient of s^1

$5 = 2A + 5B \Rightarrow$ (2) Coefficient of s^0

Sub (1) in (2)

$$5 = -2B + 5B \Rightarrow B = \frac{5}{3}$$

$$A = -\frac{5}{3}$$

$$\therefore \mathcal{L}^{-1} \frac{5}{(s+5)(s+2)} = \mathcal{L}^{-1} -\frac{5}{3}\frac{1}{s+5} + \mathcal{L}^{-1}\frac{5}{3}\frac{1}{s+2}$$

$$= -\frac{5}{3}e^{-5t} + \frac{5}{3}e^{-2t}$$

3) By Residue form :-

$$\mathcal{L}^{-1} \frac{5}{(s+5)(s+2)} = \frac{A}{s+5} + \frac{B}{s+2}$$

$$A = \frac{1}{(r-q)!} \lim_{s \to -P_i} \frac{d^{r-q}}{ds^{r-q}} (s-P_i)^r * F(s) \Big|_{s=-P_i}$$

r: Larget power

q: the power of calculation

P_i: Constant

$$\therefore A = \frac{1}{0!} \lim_{s \to -5} \frac{d^0}{ds^0} (s+5) * \frac{5}{(s+5)(s+2)} \Big|_{s=-5} = -\frac{5}{3}$$

$$B = \frac{1}{0!} \lim_{s \to -2} \frac{d^0}{ds^0} (s+2) * \frac{5}{(s+5)(s+2)} \Big|_{s=-2} = \frac{5}{3}$$

$$\therefore \mathcal{L}^{-1} \frac{5}{(s+5)(s+2)} = \mathcal{L}^{-1} -\frac{5}{3}\frac{1}{s+5} + \mathcal{L}^{-1}\frac{5}{3}\frac{1}{s+2} = -\frac{5}{3}e^{-5t} + \frac{5}{3}e^{-2t}$$

14- $\mathcal{L}^{-1} \dfrac{s+5}{(s+2)^3}$ (Part two)

① By algebric method :-

$$\mathcal{L}^{-1} \frac{s+5}{(s+2)^3} = \frac{A}{(s+2)^3} + \frac{B}{(s+2)^2} + \frac{C}{(s+2)}$$

$$\frac{s+5}{(s+2)^3} = \frac{A + B(s+2) + C(s+2)^2}{(s+2)^3}$$

$$\frac{s+5}{(s+2)^3} = \frac{A + Bs + 2B + Cs^2 + 4Cs + 2C}{(s+2)^3}$$

$$s+5 = A + Bs + 2B + Cs^2 + 4Cs + 2C$$

$0 = C$ —— ① Coefficient of s^2

$1 = B + 4C$ —— ② Coefficient of s^1

$5 = A + 2B + 2C$ —— ③ Coefficient of s^0

from ② $B = 1$

from ③ $A = 5 - 2 \Rightarrow A = 3$

$$\therefore \mathcal{L}^{-1} \frac{s+5}{(s+2)^3} = \frac{3}{(s+2)^3} + \frac{1}{(s+2)^2} + \frac{0}{(s+2)}$$

$$= \frac{3}{2} t^2 e^{-2t} + t e^{-2t}$$

② By Residue form :-

$$\mathcal{L}^{-1}\frac{s+5}{(s+2)^3} = \frac{A}{(s+2)^3} + \frac{B}{(s+2)^2} + \frac{C}{(s+2)}$$

$$A = \frac{1}{(3-3)!}\lim_{s\to -2}\frac{d^{3-3}}{d^0}\cancel{(s+2)^3}\frac{s+5}{\cancel{(s+2)^3}} = 3$$

$$B = \frac{1}{(3-2)!}\lim_{s\to -2}\frac{d^{3-2}}{ds^{3-2}}(s+2)^3\frac{s+5}{(s+2)^3} = 1$$

$$C = \frac{1}{(3-1)!}\lim_{s\to -2}\frac{d^{3-1}}{ds^{3-1}}(s+2)^3\frac{s+5}{(s+2)^3} = 0$$

$$\therefore \mathcal{L}^{-1}\frac{s+5}{(s+2)^3} = \mathcal{L}^{-1}\frac{3}{(s+2)^3} + \mathcal{L}^{-1}\frac{1}{(s+2)^2} = \frac{3}{2}t^2e^{-3t} + te^{-2t}$$

Ex $\mathcal{L}\,\dfrac{s^2+5s+2}{(s-3)^2(s+5)} = \dfrac{A}{(s-3)^2} + \dfrac{B}{(s-3)} + \dfrac{C}{s+5}$

① By algebric method :-

$$\frac{s^2+5s+2}{(s-3)^2(s+5)} = \frac{A}{(s-3)^2} + \frac{B}{(s-3)} + \frac{C}{s+5}$$

$$\frac{s^2+5s+2}{(s-3)^2(s+5} = \frac{A(s+5) + B(s-3)(s+5) + C(s-3)^2}{(s-3)^2(s+5)}$$

$$\frac{s^2+5s+2}{(s-3)^2(s+5)} = \frac{As+5A+Bs^2+2Bs-15B+Cs^2-6sC+9C}{(s-3)^2(s+5)}$$

$1 = B + C$ --- ① Coefficients of s^2.

$5 = A + 2B + 6C$ — ② Coefficients of s^1.

$2 = 5A - 15B + 9C$ — ③ coefficients of s^0.

Sub ① in ②

$3 = A - 8C$ — ④

Sub ① in ③

$17 = 5A + 24C$ — ⑤

$C = 1/32$, $A = 26/8$, $B = 31/32$

$$\mathcal{L}^{-1}\frac{s^2+5s+2}{(s-3)^2(s+5)} = \mathcal{L}^{-1}\frac{26}{8}\times\frac{1}{(s-3)^2} + \mathcal{L}^{-1}\frac{31}{32}\times\frac{1}{s-3} + \mathcal{L}^{-1}\frac{1}{32}\times\frac{1}{s+5}$$

$$= \frac{26}{8}te^{3t} + \frac{31}{32}e^{3t} + \frac{1}{32}e^{-5t}$$

2) By Residue form :-

$$A = \frac{1}{(2-2)!}\lim_{s\to3}\frac{d^0}{ds^0}\cancel{(s-3)^2}\frac{s^2+5s+2}{(s+5)\cancel{(s-3)^2}} = \frac{26}{8}$$

$$B = \frac{1}{(2-1)!}\lim_{s\to3}\frac{d}{ds^1}\cancel{(s-3)^2}\frac{s^2+5s+2}{(s+5)\cancel{(s-3)^2}}$$

$$= \frac{(s+5)(2s+5) - (s^2+5s+2)}{(s+5)^2} \Rightarrow B = \frac{31}{32}$$

$$C = \frac{1}{(1-1)!} \lim_{s \to -5} \frac{d^{\circ}}{ds^{\circ}} (s+5) \frac{s^2+5s+2}{(s+5)(s-3)^2} = \frac{1}{32}$$

$$\mathcal{L}^{-1} \frac{s^2+5s+2}{(s-3)^2(s+5)} = \mathcal{L}^{-1} \frac{26}{8} * \frac{1}{(s-3)^2} + \mathcal{L}^{-1} \frac{31}{32} * \frac{1}{s-3} + \mathcal{L}^{-1} \frac{1}{32} * \frac{1}{s+5}$$

$$= \frac{26}{8} t e^{3t} + \frac{31}{32} e^{3t} + \frac{1}{32} e^{-5t}$$

15 – $\mathcal{L}^{-1} \frac{3}{(s^2+4)(s+5)}$ (part 3)

By algebric method :-

$$\mathcal{L}^{-1} \frac{3}{(s^2+4)(s+5)} = \frac{As+B}{s^2+4} + \frac{C}{s+5}$$

$$\frac{3}{(s^2+4)(s+5)} = \frac{(As+B)(s+5) + C(s^2+4)}{(s^2+4)(s+5)}$$

$$3 = As^2 + 5As + Bs + 5B + Cs^2 + 4C$$

$0 = A + C$ —— ① coefficients of s^2

$0 = 5A + B$ —— ② Coefficients of s

$3 = 5B + 4C$ —— ③ Coefficient of s°

$$A = -\frac{3}{29}, \quad B = \frac{15}{29}, \quad C = \frac{3}{29}$$

$$\therefore \mathcal{L}^{-1} \frac{3}{(s^2+4)(s+5)} = \mathcal{L}^{-1} \frac{-\frac{3}{29}s + \frac{15}{29}}{s^2+4} + \mathcal{L}^{-1} \frac{3/29}{s+5}$$

$$= -\frac{3}{29} \mathcal{L}^{-1} \frac{s}{s^2+4} + \frac{15}{29} \mathcal{L}^{-1} \frac{1}{s^2+4} + \frac{3}{29} \mathcal{L}^{-1} \frac{1}{s+5}$$

$$= -\frac{3}{29} \cos 2t + \frac{15}{29} * \frac{1}{2} \sin 2t + \frac{3}{29} e^{-5t}$$

Solution of differential equation :-

For the differential equation :-

$$\frac{d^3y}{dt^3} + a \frac{d^2y}{dt^2} + b \frac{dy}{dt} + cy = f(t)$$

the Laplace transform is :-

$$s^3 y(s) - s^2 y(0) - s y'(0) - y''(0) + a[s^2 Y(s) - s y(0) - y'(0)] +$$
$$b[s y(s) - y(0)] + C Y(s) = F(s)$$

$$(s^3 + as^2 + bs + c) Y(s) = (s^2 + as + b) y(0) + (s+a) y'(0)$$
$$+ y''(0) + F(s)$$

$$\therefore Y(s) = \frac{F(s) * y(0)}{s^3 + as^2 + bs + c} + \frac{(s+a) * y'(0)}{s^2 + as^2 + bs + c} +$$
$$\frac{1 * y''(0)}{s^3 + as^2 + bs + c}$$

Ex Solve the diff. eq :-

$$\frac{d^3y}{dt^3} + 3\frac{d^2y}{dt^2} + 3\frac{dy}{dt} + y = t \quad , \quad y(0)=0, \ y'(0)=1, \ y''(0)=2$$

take the Laplace transform

$$s^3y(s) - s^2y(0) - s y'(0) - y''(0) + 3(s^2y(s) - sy(0) - y'(0)) + 3(sy(s) - y(0))$$

$$+ y(s) = \frac{1}{s^2}$$

$$s^3y(s) - s - 2 + 3s^2y(s) - 3 + 3sy(s) + y(s) = \frac{1}{s^2}$$

$$y(s) * [s^3 + 3s^2 + 3s + 1) - s - 2 - 3 = \frac{1}{s^2}$$

$$y(s) = \frac{s}{s^3+3s^2+3s+1} + \frac{5}{s^3+3s^2+3s+1} + \frac{1}{s^2(s^3+3s^2+3s+1)}$$

$$y(s) = \frac{s}{(s+1)^3} + \frac{5}{(s+1)^3} + \frac{1}{s^2(s+1)^3}$$

$$\therefore y(s) = \frac{s^3+5s^2+1}{s^2(s+1)^3} = \frac{A}{s^2} + \frac{B}{s} + \frac{C}{(s+1)^3} + \frac{D}{(s+1)^2} + \frac{E}{(s+1)}$$

$$A = \lim_{s\to 0} \cancel{s^2} \frac{s^3+5s^2+1}{\cancel{s^2}(s+1)^3} = 1$$

$$B = \frac{1}{1!} \lim_{s\to 0} \frac{d}{ds} \cancel{s^2} \frac{s^3+5s^2+1}{\cancel{s^2}(s+1)^3} = \frac{(s+1)^3 * (3s^2+10s) - (s^3+5s^2+1) * 3(s+1)^2}{(s+1)^6}$$

$$B = \frac{\cancel{(s+1)^2}[(s+1)(3s^2+10s) - ((s^3+5s^2+1) * 3)]}{(s+1)^{\cancel{6}}} = \frac{-3}{1} = -3$$

$$C = \lim_{s \to -1} (s+1)^3 \frac{(s^3+5s^2+1)}{s^2(s+1)^3} = 5$$

$$D = \lim_{s \to -1} \frac{d}{ds} \cancel{(s+1)^3} \frac{s^2+5s+1}{s^2 \cancel{(s+1)^3}} = \frac{s^2 \times (2s+5) - (s^2+5s+1) \times 2s}{s^4}$$

$$= \frac{\cancel{s}\,[s(2s+5) - 2(s^2+5s+1)]}{s^4} =$$

$$= \frac{-1(3) - 2(1-5+1)}{(-1)^3} = \frac{-3+6}{-1} = \frac{3}{-1} = -3$$

$$E = \lim_{s \to -1} \frac{d^2}{ds^2} (s+1)^3 \frac{s^2+5s+1}{s^2(s+1)^3} \Rightarrow E = 3$$

$$\therefore F(s) = \frac{1}{s^2} - \frac{3}{s} + \frac{5}{(s+1)^3} + \frac{3}{(s+1)^2} + \frac{3}{(s+1)}$$

$$\therefore F(t) = t - 3u(t) + \frac{5}{2} t^2 e^{-t} + 3t e^{-t} + 3e^{-t}$$

Application Circuit :-

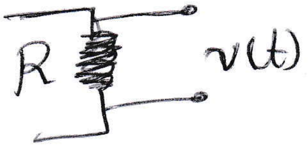
Having mastered now to obtain the Laplace transform and its inverse, we are now prepered to employ the Laplace transform to analyze.

1- transform The circuit from the time domain to S domain.

2- Solve the circuit using nodel analysis, mesh analysis, Source transformation, Super position or any circuit analysis technique with which we are familiar.

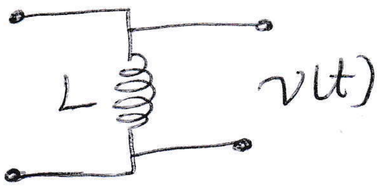
3- Take the inverse transform of the solution and thus obtain the solution in the time domain.

1- For Resistance :-



$$v(t) = R\, i(t)$$
$$V(s) = R\, I(s)$$

2- For Inductance :-



$$v(t) = L \frac{di}{dt}$$

take Laplace transform for both sides:

$$V(s) = L\,[s I(s) - I(0)] = sLI(s) - LS(0)$$

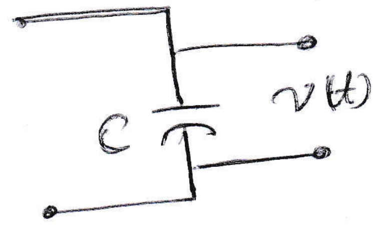
OR $$I(s) = \frac{1}{sL} V(s) + \frac{i(0)}{s}$$

3) For Capacitance

$$i(t) = C \frac{dv(t)}{dt}$$

$$I(s) = C[s V(s) - V(0)]$$

$$V(s) = \frac{1}{sC} I_s + \frac{V_0}{S}$$



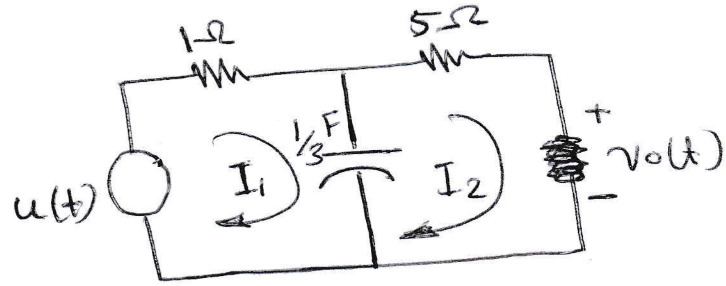
OR

$$C \, dv(t) = i(t) \, d(t)$$

$$C v(t) = \int i \, dt$$

$$V(s) = \frac{1}{sC} I(s)$$

EX Find $V_o(t)$ in the circuit in figure below, assuming Zero initial Condition.



mesh 1

$$\frac{1}{S} = \left(1 + \frac{3}{S}\right) I_1 - \frac{3}{S} I_2 \quad (1)$$

mesh 2

$$0 = \frac{-3}{S} I_1 + \left(S + 5 + \frac{3}{S}\right) I_2$$

OR $$I_1 = \frac{(S^2 + 5S + 3)/\cancel{S} \; I_2}{\frac{3}{\cancel{S}}} = \frac{S^2 + 5S + 3}{3} I_2$$

Sub (2) in (1)

$$\cancel{\frac{-3}{S}\left(\frac{S^2+5S+3}{3}\right) I_2}$$

$\frac{1}{s} = (1+\frac{3}{s})\, \frac{1}{3}(s^2+5s+3)\, I_2 - \frac{3}{s} I_2$

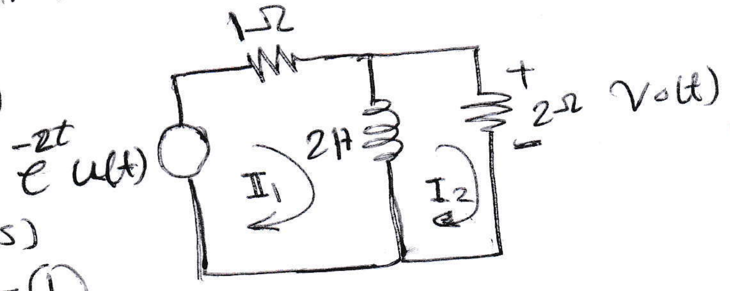
$\therefore\ I_2 = \frac{3}{s^3+8s^2+18s}$, $V_o(s) = s\, I_2$.

$\therefore\ v_o(s) = \frac{3}{s^2+8s+18} = \frac{3}{s^2+8s+16+2}$

$\therefore\ v_o(s) = \frac{3}{(s+4)^2+2}$

$\therefore\ v_o(t) = \frac{3}{\sqrt{2}}\, e^{-4t} \sin\sqrt{2}\, t$

<u>EX</u> Find $v_o(t)$ in the circuit below :-



$e^{-2t} u(t) = I_1 + L\frac{d}{dt}(I_1 - I_2)$

$\frac{1}{s+2} = I_1(s) + 2s I_1(s) - 2s I_2(s)$ —— ①

$0 = L\frac{d}{dt}(I_2 - I_1) + R\, I_2$

$0 = 2s\, I_2(s) - 2s\, I_1(s) + 2\, I_2(s)$ —— ②

$I_2(s)\,(s+1) = \cancel{2}\, s\, I_1(\, I_1(s)$

$I_2(s) = \frac{s}{s+1} I_1(s)$ —— ③

$\frac{1}{s+2} = I_1(s) + 2s\, I_1(s) - 2s * \frac{s}{s+1} I_1(s)$

$$\frac{1}{s+2} = I_1(s)\left[1+2s - \frac{2s^2}{s+1}\right]$$

$$\frac{1}{s+2} = I_1(s)\left[\frac{s+1+\cancel{2s^2}+2s-\cancel{2s^2}}{s+1}\right]$$

$$\frac{1}{s+2} = I_1(s)\left[\frac{3s+1}{s+1}\right]$$

$$I_1(s) = \frac{\frac{1}{s+2}}{\frac{3s+1}{s+1}} = \frac{s+1}{(s+2)(3s+1)}$$

sub in (3) to get $I_2(s)$

$$I_2(s) = \frac{s}{\cancel{s+1}} \times \frac{\cancel{s+1}}{(s+2)(3s+1)}$$

$$I_2(s) = \frac{\frac{1}{3}s}{(s+2)(s+\frac{1}{3})} = \frac{A}{s+2} + \frac{B}{s+\frac{1}{3}}$$

$$A = \lim_{s\to -2} \cancel{(s+2)} \frac{\frac{1}{3}s}{\cancel{(s+2)}(s+\frac{1}{3})} = \frac{-\frac{2}{3}}{-\frac{5}{3}} = \frac{2}{5}$$

$$B = \lim_{s\to -\frac{1}{3}} \cancel{(s+\tfrac{1}{3})} \frac{\frac{1}{3}s}{(s+2)\cancel{(s+\frac{1}{3})}} = \frac{-\frac{1}{9}}{\frac{5}{3}} = -\frac{1}{15}$$

$$I_2(s) = \frac{\frac{2}{5}}{s+2} - \frac{\frac{1}{15}}{s+\frac{1}{3}}$$

$$v_o(s) = 2I_2(s) \Rightarrow \frac{\frac{4}{5}}{s+2} - \frac{\frac{2}{15}}{s+\frac{1}{3}}$$

$$v_o(t) = \frac{4}{5}e^{-2t}u(t) - \frac{2}{15}e^{-\frac{1}{3}t}u(t).$$

Transfer function $H(s)$

It is the ratio of the output assuming all initial condition are zero.

$$H(s) = \frac{Y(s)}{X(s)} \quad , \quad H(s) = \text{Voltage gain} = \frac{V_o(s)}{V_i(s)}$$

$$H(s) = \text{Current gain} = \frac{I_o(s)}{I_i(s)}$$

$$H(s) = \frac{V(s)}{I(s)} \quad \text{" impedance"}$$

$$H(s) = \frac{I(s)}{V(s)} \quad \text{"admittance"}$$

$$Y(s) = H(s) * X(s)$$

$$h(t) = \mathcal{L}^{-1} H(s)$$

impedance transfer function

EX the output of linear System is $y(t) = 10e^{-t}\cos 4t$ $u(t)$, when the ~~impedance~~ output is $x(t) = e^{-t}u(t)$. Find the transfer function of the system and its impulse response :-

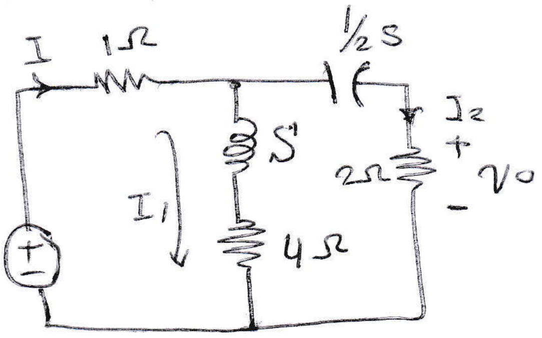
$$x(t) = e^{-t}u(t) \Rightarrow X(s) = \frac{1}{s+1}$$

and $$y(t) = 10e^{-t}\cos 4t\, u(t) \Rightarrow Y(s) = \frac{10(s+1)}{(s+1)^2+4^2}$$

$$\therefore H(s) = \frac{Y(s)}{X(s)} = \frac{10(s+1)^2}{(s+1)^2+4^2}$$

EX Determine the transfer function.

1) $H(s) = \dfrac{V_o(s)}{I_o(s)}$, 2) $H(s) = \dfrac{I_1(s)}{I_2(s)}$ for figure below.



① By current division

$$I_2 = \frac{(s+4)}{s+4+2+\frac{1}{2s}}$$

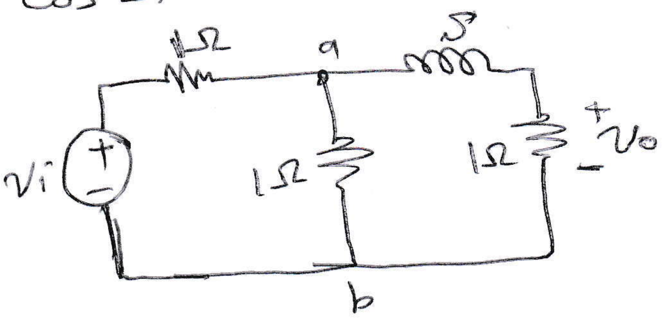
But $$V_o = 2 I_2 = \frac{2(s+4) I_o}{s+6+\frac{1}{2s}}$$

$$\therefore H(s) = \frac{V_o(s)}{I_o(s)} = \frac{4s(s+4)}{2s^2+12s+1}$$

② $$I_1 = \frac{I_o(2+\frac{1}{2s})}{s+4+2+\frac{1}{2s}} = \frac{I_1}{I_o} = \frac{4s+1}{2s^2+12s+1}$$

$$\therefore H(s) = \frac{I_1(s)}{I_o(s)} = \frac{4s+1}{2s^2+12s+1}$$

EX For the s-domain circuit find ⓐ the transfer function (T.F.) $H(s) = \dfrac{V_o}{V_i}$ ⓑ the impulse response ⓒ the response when $v_i(t) = u(t)$ ⓓ the response when $v_i(t) = 8\cos 2t$.



a) using voltage division

$$V_o = \frac{1}{s+1} V_{ab} \quad \text{—} (1)$$

$$V_{ab} = \frac{1 // (s+1)}{1 + 1//(s+1)} V_i = \frac{(s+1) / (s+1)}{1+\frac{(s+1)}{(s+1)}} V_i$$

$$V_{ab} = \frac{s+1}{2s+3} V_i \quad \text{———} \; (2)$$

sub (2) in (1) we obtain

$$V_o = \frac{V_i}{2s+3} \Rightarrow \frac{V_o(s)}{V_i(s)} = \frac{1}{2s+3}$$

b) $H(s) = \frac{1}{2} \frac{1}{s+\frac{3}{2}}$

$H(t) = \frac{1}{2} e^{-\frac{3}{2}t} u(t)$

c) when $v_i(t) = u(t) \Rightarrow V_i(s) = \frac{1}{s}$

$$V_o(s) = H(s) * V_i(s) = \frac{1}{2s(s+\frac{3}{2})}$$

$$= \frac{A}{s} + \frac{B}{s+\frac{3}{2}} \Rightarrow A = \frac{1}{3}, \; B = -\frac{1}{3}$$

$$\therefore V_o(s) = \frac{1}{3}\left(\frac{1}{s} - \frac{1}{s+\frac{3}{2}}\right) \Rightarrow V_o(t) = \frac{1}{3}\left(1 - e^{-\frac{3}{2}t}\right) u(t)$$

d) when $v_i(t) = 8\cos 2t$, then $V_i(s) = \frac{8s}{s^2+4}$, and

$$V_o(s) = H(s) * V_i(s) = \frac{4s}{(s+\frac{3}{2})(s^2+4)}$$

$$= \frac{A}{s+\frac{3}{2}} + \frac{Bs+C}{s^2+4} \Rightarrow A = -\frac{24}{25}, \; B = \frac{24}{25}$$

$$C = \frac{64}{25}$$

$$V_o(s) = \frac{-24/25}{s+\frac{3}{2}} + \frac{24}{25} \frac{s}{s^2+4} + \frac{32}{25} * \frac{2}{s^2+4}$$

$$\therefore v_o(t) = \frac{24}{25}\left(-e^{-\frac{3}{2}t} + \cos 2t + \frac{4}{3} \sin 2t\right) u(t)$$